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ELECTRICAL EQUIPMENT FOR THE EXPERIMENTAL STUDY OF THE DYNAMICS OF FLUIDS

By Carlo Ferrari

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SUMMARY

This report contains the description of electric anemometers and their application to the study of turbulent fluid flows, of electric tanks for the realization of the analogies between electrology and aerodynamics and their application to the study of varied technical problems, and lastly of the electric condenser type dynamometer and its application to the prediction of the aerodynamic forces on wing and airplane models in wind-tunnel tests and and in controlled and spontaneous rotations.

INTRODUCTION

The electrical devices employed for the experimental study of the dynamics of fluids can be divided into two large categories: The first comprises all the devices for analyzing the structure of the field of motion, or, if speed is involved, the path of the fluid particle, in short, the kinematic elements of the phenomenon, while the other embodies the equipment for recording the dynamic actions, that is, of the forces which the flow exerts, say, on obstacles in relative motion with respect to it.

In the description of the principal apparatus of either kind and the discussion of the most important results obtainable therewith, brief mention is made of the best known types either with special attention to those which are original or the more particular problems involved in their application.

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*"Sistemazioni elettriche per lo studio sperimentale della dinamica dei fluidi." Societa Italiana per il Progresso delle Scienze, Roma, 1938 - XVI.

ELECTRIC ANEMOMETRY AND ITS APPLICATION

Electric Anemometer for Measuring the Speed

We begin with the equipment of the first category outlined above, and, in particular, examine the electric speed-measuring devices.

The principle upon which the electric anemometer is based has been known for a long time, but the most important and most interesting applications are of relatively recent date: a metal wire heated with gas assumes a temperature which, other things being equal, is a function of the speed of the flow involved, and since the electric resistance of the wire itself changes with its temperature, it is easy to understand that the characteristics of the circuit of which the wire forms a part, are dependent upon the speed of the air striking it, and that consequently it is possible to measure the speed and direction of air flow. The application of this principle, originally announced by Professor U. Bordoni in Italy (reference 1) almost simultaneously with a number of authors abroad (reference 2), conforms in general outlines to the diagrams shown in figure 1: The anemometer wire F forming one arm of a Wheatstone (or Thompson) bridge, the ratio arms being of known resistance fixed and constant with temperature, while the diagonal contains a galvanometer or a very sensitive voltmeter; if the bridge is in balance when the speed of flow is zero, an air current striking F cools it, and causes an unbalance which is read on the galvanometer G. The two diagrams differ merely in the method by which balance is restablished in the bridge; in hook-up I the so-called "constant resistance" method is employed: The intensity i is regulated by the variable resistance R_1 in the feed circuit, so that the temperature θ of the wire F is always the same whatever the speed of the flow. Hence the resistance r of wire F always assumes the value that corresponds to the conditions of bridge balance. As a result the intensity i in the feed circuit is a function of the speed V of the flow $i = f_1 (V)$, which is measured by a potentiometer method as indicated in figure 1.

However, the bridge balance can also be restored by compensating the variation in the resistance of F with a variation equal and opposite placed in series with F on the same side. By this method the total resistance of the circuit remains constant and therefore, since the electromotive force generated by a storage battery is also invariable, the current flowing through wire F remains constant, while the resistance r of wire F, being a function of the speed $r = f_2(V)$, must be secured by calibration. This is called "constant current" method.

Sensitivity of the Electric Anemometer

Irrespective of the method used for restoring the bridge balance, the form of the function which connects the electric quantity to be measured with the speed of the flow depends upon the angle formed by the direction of the wire with that of the latter in accordance with the law connecting the speed with the heat transmission Q between wire and flow. With wire normal to the air flow we get, according to King's formula (reference 3) and as confirmed qualitatively in tests by many other experimenters such as Van der Hegge Zyinen (cf. reference 1), and Bailey and Simmons (R. and M. No. 1019)

$$Q = (\theta - \theta_0)[b\sqrt{V} + c]\,l$$

with wire parallel to air flow the result is, according to Bailey-Simmons:

$$Q = (\theta - \theta_0)[b_1 V + c_1\theta + c_2]\,l$$

where θ_0 is temperature of flow, and b, c, b_1, c_1, c_2, constants depending upon wire material, upon length of wire l, and its diameter.

Since, under the conditions of the speed it should afford $0.239\, i^2 r = Q$ (i in amp., r in ohms, Q in calories), we have

$$\left.\begin{aligned} i^2 &= \left(1 - \frac{r}{r_0}\right)F(V) \\ i^2 &= \left(1 - \frac{r}{r_0}\right)F_1(V,\ r) \end{aligned}\right\} \text{wire normal to air flow and parallel to air flow, respectively} \qquad (1)$$

with

$$\left.\begin{aligned} F\ (V) &= \frac{b\sqrt{V}}{0.239\alpha r_o} + \frac{c}{0.239\alpha r_o} && (\alpha \text{ temperature coefficient of } F) \\ F_1(V,r) &= \frac{b_1\ V}{0.239\alpha r_o}\quad \frac{c_1\ r}{0.239\alpha^2 r_o{}^2} && (r_o \text{ resistance to temperature } \theta_o) \\ &+ \frac{+r_o c_1 \alpha\theta_o + r_o\alpha c_2 - c_1 r_o}{0.238\alpha^2 r_o{}^2} \end{aligned}\right\} \quad (2)$$

where r and r_o represent the resistance per unit length of wire.

Formulas (1) and (2) are forthwith applicable to the sensitivity of the system under the various conditions in question. The "constant resistance" method affords

$$\left.\begin{aligned} \frac{\partial i}{\partial V} &= \frac{F'}{F}\ i = \frac{i}{2V + 2\,\frac{c}{b}\sqrt{V}} && \text{wire normal to air flow} \\ \frac{\partial i}{\partial V} &= \frac{F'_1 V}{F_1}\ i = \frac{i}{V + \frac{c_1}{b_1}\,\theta + \frac{c_2}{b_1}} && \text{wire parallel to air flow} \end{aligned}\right. \quad (3)$$

while the "constant current" method gives

$$\left.\begin{aligned} -\frac{\partial r}{\partial V} &= \frac{2F'r}{F}\left(\frac{r}{r_o} - 1\right) = \frac{\alpha(\theta - \theta_o)r}{V + \frac{c}{b}\sqrt{V}} && \text{wire normal to air flow} \\ -\frac{\partial r}{\partial V} &= \frac{2\ F'_1 Vr}{F_1\left(\frac{r_o}{r - r_o} + \frac{rF'_{1r}}{F'}\right)} && \text{wire parallel to air flow} \\ &= \frac{2}{V + \frac{c_1}{b_1}\,\theta + \frac{c_2}{b_1}}\ \frac{r}{\frac{r_o}{r - r_o} + \frac{rF'_{1r}}{F_1}} \end{aligned}\right\} \quad (4)$$

With the foregoing formulas the values of δi

(in ma) and of δr (in 10^{-3} ohm per cm of length of wire) corresponding to a 10 centimeter-per-second change in speed δV were computed at $V = 1$ meter per second and $V = 10$ meters per second, with a platinum wire of 0.027 millimeter diameter and 78 millimeters length. The Bayley and Simmons values for b, c, b_1, c_1, and c_2 are:

$$b = 3.98 \times 10^{-7};\ c = 3.15 \times 10^{-6};\ b_1 = 1.8 \times 10^{-9};$$

$$c_1 = 2.46 \times 10^{-9};\ c_2 = 3.3 \times 10^{-6}$$

where V is speed (cm/sec). For a temperature coefficient of $\alpha = 0.0024$, the results are:

By Constant Resistance Method

	Wire normal to air flow		Wire parallel to air flow	
δ 10^{-3} amp For $\delta V = 10$ cm/sec	a) (With $\theta - \theta_0 = 100°$)1.07 b) (with $\theta - \theta_0 = 100°$)1.7	a) 0.228 b) 0.374	a) 0.32 b) 0.192	0.112 0.167
V (m/s)	1	10	1	10

By Constant Current Method

	Wire normal to air flow		Wire parallel to air flow	
δ 10^{-3} ohm For $\delta V = 10$ cm/sec	a) (With i=30 mm) 11.5 b) (With i=60 mm)162	0.83 4.7	a) 5.2 b) 27.8	2.28 19.8
V (m/s)	1	10	1	10

Examination shows a loss of sensitivity with increasing air speed. Further, it will be seen that the constant resistance method is more sensitive than the constant-current method at low speeds (equation (4)), but that the loss of sensitivity with the speed is greater by the former than by the latter method.

Ratio of Sensitivity

First method – wire normal	4.6	Wire parallel	1.16
Second method – wire normal	$\begin{cases}14\\34\end{cases}$	Wire parallel	$\begin{cases}2.28\\1.4\end{cases}$

For the two values of i in question

So, with wire used normally it is possible to obtain greater sensitivity, while with wire used parallel the loss of sensitivity is less rapid at increasing speed V; hence, for sufficiently high V values the second arrangement is ultimately more sensitive.

To obviate the inconvenience of loss of sensitivity of the anemometer, which is very rapid on very sensitive devices, various schemes have been devised (reference 4), but the results are not at all satisfactory. In this connection it should be noted that the importance of research for the purpose of obtaining a linear relationship between the instrument reading and the air speed is based, not so much on the possibility of extending in this manner the field of velocity for which such instrument itself can be used, as on the greater reliability of the results secured with it when applied to the measurement of the speed fluctuations, as produced, for instance, in turbulent flow.

Anemometer Used for Measuring the Speed and Direction of Air Flow

The determination of the speed implies measuring in intensity and direction with respect to a suitable system of reference axes. This is achieved in simple and accurate manner with three or four wires grouped symmetrically about an axis to form a triangular or square pyramid having a vertex angle of about 10°, as indicated in figure 2. The wire pyramid can be made to rotate about a vertical axis so as to assume any position in space; the insertion of the wires into the circuit is seen in the diagrammatic sketch of the circuit (fig. 3). Circuit (a) is used for measuring the direction of the speed: wire F_2 or F_3 is inserted into one arm of a Wheatstone bridge of which the adjacent arm is formed by wire F_1. If the wires are not symmetrically inclined with respect to the direction of the air flow and if the bridge was in balance at zero speed, the bridge will be out of balance because of the difference in cooling of the wires. To restore the balance the pyramid must be rotated about O so that its axis is along the direction of the flow. Circuit (b) is used for measuring the speed of flow. The two wires F_1 and F_2 placed in series form the same arm of a Wheatstone bridge and the speed is measured by either one of the two methods explained above. The sensitivity of the instrument with respect to direction of speed depends upon radius and length of wires, and angle θ along which they are arranged. From the data relative to those considered first and for $\theta = 10^\circ$ (corresponding to the conditions of optimum sensitivity for wind along the direction of the axis of the pyramid) follows the difference in resistance δr of the two wires for an inclination of φ_3 degrees of the speed above this axis, $\delta r = 200 \times 10^{-3} \varphi$ ohms practically independently of speed of air flow.

If the flow is two-dimensional the direction can also be measured by a method suggested by Thomas (reference 5) and developed by Burgers (reference 6). This method also makes use of two wires F_1 and F_2, but instead of being set at an angle they are placed parallel and very close together. If the air is normal to the plane of the two wires, both are cooled alike; hence they offer the same resistance and, when inserted into the opposite arms of a Wheatstone bridge, do not put it out of balance. If, on the other hand, the wind, although it is normal to the plane of both wires, is inclined with respect to the plane of the wires, the unsymmetry of the field of velocity about them does not cause a difference in cooling. The result is an unbalance of the bridge which can be counteracted only by a rotation of the mechanism supporting the anemometric wires so that the plane of the wires is normal to the flow. According to Burgers (loc. cit.) the sensitivity of this instrument with platinum-iridium wires of 12.9 millimeters length and 0.015 diameter, spaced 0.052 millimeter apart, at a speed of 1000 centimeters per second, is:

$$\left(\frac{\partial r}{\partial v}\right)_{i=const} = 6.4 \times 10^{-3} \text{ ohm/cm/sec}; \quad \delta r = 14.3 \times 10^{-3} \varphi^\circ \text{ ohm}$$

The difference in resistance δr of the wires as a result of the slope of the wind depends in a small measure upon its speed.

The minor disturbances which the electric anemometer produces in the field of velocity about a test point, the convenience of application, the great sensitivity at low speed together with a minimum of inertia, render the anemometer particularly suitable for wake exploration, for obtaining the field of flow in boundary layers, etc., and in fact, among the more recent applications, it suffices to mention A. Fage - L. F. G. Simmons (Phil. Trans. 225, 1925), "An Investigation of the Flow of Air around an Aerofoil of Infinite Span;" L. F. Simmons - A. F. C. Brown, "An Experimental Investigation of Boundary Layer Flow, with Special Reference to Methods of Detecting Transitional Regions;" R. and M. no. 1547, Nov. 1934; Van der Hegge Zyinen, "Measurement of the Velocity Distribution in the Boundary Layer along a Plane Surface (Thesis Delft, 1924), etc.

ELECTRIC ANEMOMETER FOR MEASURING THE FLUCTUATIONS OF AIR-SPEED IN TURBULENT FLOW

Errors Due to Thermal Lag

The application of hot-wire anemometers, however, of first interest and importance for the purposes of study of the dynamics of fluids is without doubt that used in the determination of rapid and irregular fluctuations of air-speed in turbulent flow where any other type of anemometer is absolutely unsuitable.

The feasibility of making measurements of this kind with a hot-wire anemometer is readily apparent from a glance at figure 1, by visualizing the turbulence isotropic and wire F placed normal to the air flow. The fluctuations of the airspeed induce similar variations in the resistance of the wire. Hence, every fluctuation in speed defines a variation in drop of potential across the ends of F, or, which amounts to the same thing, across the arms C and D of the bridge. Hence, if this oscillating drop of potential, say by ΔV, is applied to a thermic voltmeter or to a cathode-ray oscillograph the average square of the fluctuations or an oscillogram can be obtained which defines the variations of ΔV with respect to time, no matter how fast or irregular it may be. But, in order to proceed from the records of the instrument to the fluctuations of speed, or at least to its dharacteristic magnitude, it is necessary that ΔV be proportional at any instant to the speed fluctuations causing it. As first consequence of these conditions it appears at once that the length of the anemometer wire must be very small compared to what Taylor calls the "average size of the eddy" in direction parallel to the turbulence wire which takes part in the turbulent diffusion. In other words, where $R(y)$ denotes the correlation coefficient between the velocity fluctuations V at two points in the same plane normal to V, distance y apart, and

$$L = \int_0^\infty R\,(y)\,d\,y$$

wire length l must be small compared to L, whence the velocity fluctuations for all l can be considered in complete correlation with each other.

In the opposite case a correction factor is obtained, as demonstrated by Dryden (reference 7), because of the mean square average of the velocity fluctuations in the specific case of isotropic turbulence in question.

$$K = \frac{1}{\sqrt{2\int_0^l \left(1 - \frac{y}{l}\right) R(y)dy}} \tag{5}$$

The inference from the rapid drop of $R(y)$ with y is that wire lengths of slightly more than 2 millimeters might be difficult to use (Ziegler, reference (8) used wires of 1 mm length!). It is obvious therefore that, to assure adequate sensitivity of the instruments, the diameter must be very small. Even so, the temperature θ of the wire does not assume at every instance the value θ_1 which would correspond, under speed conditions, to the actual value of the airspeed, but for small fluctuations in comparison to the average the temperature is related to it by the equation

$$\frac{d\theta}{dt} = \frac{1}{M}\,(\theta - \theta_1) \tag{6}$$

where $$M = \frac{4.2\ \delta\ S^2\ c(\theta - \theta_0)}{\rho\ i^2} \tag{7}$$

δ being the density of material of wire F, c its specific heat value, ρ the resistivity; θ the mean temperature value of the experimental wire, S the area of cross section, and θ_0 again the temperature of the flow. Dryden (reference 9) found from equation (7) that, when $\theta_1(t)$ is expanded in a Fourier series, the effective temperature value θ at any instant is also expressed by means of a Fourier series as harmonics of the same frequency as those of θ_1, but the amplitudes e and the phases of the harmonics themselves are not equal to the values corresponding to those of θ_1; although the amplitude of the harmonics of frequency $np/2\pi$ for θ is equal to the amplitude a_n of the corresponding harmonic of θ_1 divided by $\sqrt{1 + n^2\ p^2\ M^2}$, while its phase is equal to the phase of the latter reduced by $(\tan^{-1})\ n\ p\ M$. The result is that the constant M characterizes the thermal lag of the wire, and since for small fluctuations of θ it may be considered that proportional fluctuations of the speed of air flow correspond to them, it is deduced that any fluctuation in V is reproduced with a distortion which is so much greater as the said variation is richer in harmonics at high frequency and relatively high amplitudes. In fact, the turbulent fluctuations have no specific frequency nor can they be split up as for any periodic motion, but it is known from the impossibility of the anemometer to faithfully follow a uniform and continuous variation of the air, if it is very rapid, that it is also impossible to reproduce accurately a rapid non-uniform pulsation. It has been possible, however, to give another and perhaps more comprehensive expression to the foregoing results, one which adheres more closely to the turbulent phenomena. With λ denoting the "wave length" of a pulsation which appears in the turbulent fluctuation the mean-square average $\overline{u'^2}$, with Dryden (reference 10), can be expressed by

$$\overline{u'^2} = \int_0^\infty \overline{u^2}_\lambda\ d\ \lambda \tag{8}$$

where $\overline{u^2}_\lambda$ is a function of λ. The mean-square average of the pulsations measured by the anemometer is instead, in accordance with the foregoing.

$$\overline{u'_1{}^2} = \int_0^\infty \frac{\overline{u^2_\lambda}\, d\lambda}{1 + M^2U^2/\lambda^2}$$

(if U is the speed of the flow) so that the known theorem of the average is expressed with

$$\overline{u'_1{}^2} = \int_{\lambda_1}^\infty \overline{u^2_\lambda}\, d\lambda \tag{9}$$

where λ_1 is a suitable value of λ. According to (9) it appears that the effect of the thermal lag of the wire is to suppress any pulsations the wavelength of which is inferior to a given value of λ_i. Hence the error of the instrument is

$$\Delta = \overline{u'^2} - \overline{u'_1{}^2} = \int_0^{\lambda_1} \overline{u^2_\lambda}\, d\lambda \tag{10}$$

and, if λ_1 is small enough,

$$\Delta = \left(\frac{d\,\overline{u^2_\lambda}}{d\lambda}\right)_{\lambda=0} \lambda_1$$

Now, if we accept for $\overline{u^2_\lambda}$ the function defined by Dryden (reference 10)

$$\overline{u^2_\lambda} = \frac{\overline{u'^2}}{L} \frac{1}{\left[h\left(\frac{\lambda}{L}\right)^{-\frac{1}{2}} + k\left(\frac{\lambda}{L}\right)^{m/2}\right]^2}$$

where L is given by equation (5) and records the scale of turbulence, while h, k, and m are suitable constants

$$\left(\frac{d\,\overline{u^2_\lambda}}{d\lambda}\right)_{\lambda=0} = \frac{\overline{u'^2}}{L}\frac{1}{h^2} = a\,\frac{\overline{u'^2}}{L}$$

with $a = \dfrac{1}{h^2}$ = constant is obtained.

Therefore, let $$\frac{\Delta}{\overline{u'^2}} = a\,\frac{\lambda_1}{L}$$

On the other hand, λ_1 certainly is dependent upon M, since it cancels with it, and because it signifies the wavelength it must likewise contain the turbulent velocity $\sqrt{\overline{u'^2}}$. Hence $\lambda_1 = b M u$, b being a dimensionless constant (since M has the dimensions of time) and $u = \sqrt{\overline{u'^2}}$. Hence

$$\frac{\Delta}{\overline{u'^2}} = c \frac{Mu}{L}$$

with c constant. But, according to Taylor (reference 11) and Karman (reference 12), it is

$$\frac{\epsilon^2 u}{\nu L} = A^2$$

where ν is the coefficient of viscosity of the fluid at A, a constant, and ϵ is the size of the smallest eddies taking part on the turbulence, defined by

$$\frac{1}{\epsilon^2} = \frac{1}{2} \left(\frac{d^2 R(y)}{d\ y^2}\right)_{y=0}$$

where R(y) is the correlation coefficient between the speed fluctuations originally in question; whence follows

$$\frac{\Delta}{\overline{u'^2}} = c_1 \nu \frac{M}{\epsilon^2}$$

or, with d as wire diameter and the formula substituting for M, since i^2 is approximately proportional to S:

$$\frac{\Delta}{\overline{u'^2}} = A \left(\frac{d}{\epsilon}\right)^2 \tag{11}$$

or, in other words, since the error due to the thermal lag of the hot wire is small, the wire diameter itself must be very small in comparison to the sizes of the smallest eddies participating in the turbulent diffusion. The same formula indicates how the diameter of the wire should be changed in similar fields to obtain the same error, and how, in the same field, the error varies acccording to the position of the point in which the turbulent fluctuations are measured. It is seen particularly that, since ϵ decreases (reference 13) on approaching the walls, the error committed with a given anemometer increases, discounting any other circumstances. And since, as is shown by the more recent experiments (reference 7), the radius of curvature of the diagram defining $R(y)$ as function of y is quite small for $y = 0$, the reason for the extreme thinness of the wires used in measurements of this kind is readily apparent. It is sufficient to state that Taylor in his recent experiments (reference 11) used platinum wires of 1 millimeter length and 0.0025 millimeter diameter! But even this thinness is insufficient in the majority of cases to assure a negligible value of the error.

Compensating Circuits for Reducing the Thermal Lag in the Anemometer

Fortunately the reduction of Δ can be achieved by a method other than action on the wire diameter, as indicated by Dryden (reference 9); it is based on the following principle: Assume that P_1 and P_2 are two points directly connected with the arms C and D of a Wheatstone bridge comprising the hot wire (fig. 1) and that P_1 and P_2 are connected to a resistance R_0. For the fluctuations of the resistance r of the anemometer wire produced by the harmonic oscillations of the air speed of frequency $p/2\pi$, there is generated between P_1 and P_2 a difference in potential and variable by the same law of variation of r. If these same points are connected with a circuit comprising a resistance R_1 and an inductance L of the resistance R_2, the variable current flowing through the said current will be

$$i = \frac{e}{R_1 + R_2 + i\,p\,L}$$

hence the difference in potential between P_2 and P is $e_1 = (R_2 + i\,p\,L)j$ (j = intensity of current). From this follows the ratio between e_1 and e_2 at:

$$\frac{e_1}{e_2} = \frac{R_2 + i\,p\,L}{R_1 + R_2 + i\,p\,L} = \frac{\sqrt{1 + p^2L^2/R_2^2}}{\sqrt{\left(\frac{R_1}{R_2} + 1\right)^2 + \frac{p^2L^2}{R_2^2}}}\, e^{i\theta}$$

$$\theta = (\tan^{-1}) \frac{pL/R_2}{1 + R_2/R_1 + p^2L^2/R_2R_1}$$

The ratio of amplitudes E_1 and E_2 of e_1 and e_2 therefore is

$$\frac{E_1}{E_2} = \frac{\sqrt{1 + p^2L^2/R_2^2}}{\left(1 + \frac{R_1}{R_2}\right)\sqrt{1 + \frac{p^2L^2}{R_2^2\left(1 + \frac{R_1}{R_2}\right)^2}}}$$

hence it is assumed $L/R_2 = M$ since

$$\frac{E_2}{E'_2} = \frac{1}{\sqrt{1 + M^2p^2}} \tag{11}$$

if E'_2 is the amplitude defining zero thermal lag

$$\frac{E_1}{E'_2} = \frac{1}{\left(1 + \frac{R_1}{R_2}\right)\sqrt{1 + M'^2p^2}} \tag{12}$$

is obtained, where

$$M' = \frac{M}{1 + R_1/R_2} \tag{13}$$

Now $\frac{R_1}{R_2} + 1$ is a constant factor; it only reduces the amplitudes of any harmonics existing in air fluctuations in constant manner; hence, if compared to (12) with (11), it follows forthwith that by arranging the ciccuit (L, R_1, R_2) and taking the potential drop across P_2 and P, the thermal lag from M to M' can be reduced, that is, in the ratio $\frac{R_2}{R_1 + R_2}$, but the sensitivity is reduced at at the same time in the same ratio. The error in phase however, is not reduced in the same measure and, in fact, the phase displacement given in advance by the compensating circuit is

$$\theta = \frac{p\ M}{1 + R_2/R_1 + p\ M\ L/R_1}$$

while the lag due to thermal lag is $\theta_1 = p\ M > \theta$; hence there remains a residuary error

$$p\ M \frac{\frac{R_2}{R_1} + \frac{p\ M\ L}{R_1}}{1 + p\frac{M\ L}{R_1} + \frac{R_2}{R_1}}$$

which increases with the frequency $\frac{p}{2\pi}$ of the fluctuations. It is important to note, however, that, in accordance with the results of the preceding investigation such an error in phase has no effect on the determination of the mean-square average of the turbulent oscillations.

A compensating circuit similar to the one just described was used by Ziegler (reference 14). It differed from Dryden's circuit in that he arranged a capacitance C (fig. 4a) on R_1 in shunt instead of in series with R_1.

Amplifying Circuit for Studying Turbulent Flows

Thus it is easy to understand the possibility of reduction to a minimum of the thermal lag of the hot wire, a reduction which finds its limitation in the impossibility of amplifying the voltage fluctuations between P_1 and P_2 beyond a certain value, without incurring other equally serious causes of error because of excessive amplification. At the same time the necessity for resorting to such amplification is recognized. Amplification is further dictated by the short wire length and the relatively small velocity fluctuations ordinarily produced. Every hot wire anemometer is therefore fitted with a suitable amplifying circuit. Such amplifier should meet the following requirements: it must give a high degree of amplification (the voltage fluctuations produced in the wire are of the order of millivolts and ultimately reduced to less than a thousandth part of the compensation! The ratios of multiplication required are therefore of the order of 10^6 without producing distortion. The two readings should be independent of the frequency of fluctuations and the multiplication ratio should be constant with time. These conditions, which are somewhat conflicting, make the design of an amplifier rather difficult, and the resulting wiring diagram appears quite complicated, as exemplified in Dryden's circuit (fig. 5) (reference 15) and Ziegler's hookup (fig. 6) (reference 14). The latter also indicates the device for the experimental determination of the instrument's sensitivity under high-frequency fluctuations. It comprises an alternating-current generator of low potential with frequency variable from a few to 15,000 cycles. This alternating current is superimposed on the constant current heating the wire and, on heating the wire, causes a variation of its resistance in perfectly similar manner to that corresponding to the fluctuations of the airspeed. From the comparison of the oscillograms secured at the amplifier output with those ascribed to the hot wire follows its capacity to reproduce oscillations of high frequency; hence an indication of its thermal lag. The compensating circuit R_1, R_2, C, in Ziegler's diagrammatic sketch is mounted ahead of the amplifier; whereas Dryden's circuit (R_1, R_2, L) appears inserted in an intermediate stage of the amplifier itself. There is no doubt but that the latter hookup is less advantageous, since the oscillations of the tension generated in the hot wire are already attenuated in the compensator before reaching the first amplifying stage; the disturbances produced in it which are superposed on them have a relatively great weight, which is not lowered in the succeeding stages inasmuch as the latter are amplified in the same ratio as the former.

Determination of the Correlation Coefficient of the Speed Fluctuations in Isotropic Turbulence

In isotropic turbulence studies the determination of the amplitude of turbulent fluctuations is not sufficient for a characterization of the turbulence, but requires, in addition, the extent of variation in the correlation coefficient $R(y)$ between the fluctuations at two points distance y apart, which can be achieved with the electric anemometer by arranging in a plane normal to the mean current two identical wires traversed by an electric current of equal intensity and applying the sum of or the difference of potential drop produced in them to an amplifier with an identical compensating circuit, as, for example, that of Dryden (fig. 5) according to the diagrammatic sketch (fig. 7), which illustrates the circuits used by Dryden, Schubauer, etc., in turbulence studies in wind tunnels. Then if e_1 and e_2 are the instantaneous potential drop across the two wires and u_1 and u_2 the speed fluctuations in the same instant in correspondence with the two wires and if the thermal lag is sufficiently small, it affords (Dryden, Schubauer, etc., loc. cit.):

$$e_1 + e_2 = k(u_1 + u_2); \quad e_1 - e_2 = k(u_1 - u_2)$$

where the reading H_1 and H_2 by thermal type milliammeter in the output of the amplifier can be expressed with

$$H_1 = k\overline{(u_1 + u_2)^2} = k(\overline{u_1^2} + \overline{u_2^2} + 2\,\overline{u_1 u_2})$$

$$H_2 = k(\overline{u_1^2} + \overline{u_2^2} - 2\,\overline{u_1 u_2})$$

(the bars signify average values); whence

$$\frac{H_1 - H_2}{H_1 + H_2} = \frac{\overline{u_1 u_2}}{\overline{u^2}}$$

since obviously $\overline{u_1^2} = \overline{u_2^2} = \overline{u^2}$.

Taylor, on the other hand, determined $R(y)$ by passing the currents generated in the arms of two Wheatstone bridges - each containing a hot wire, suitably amplified and compensated - in the coils of an electrodynamometer; the resulting deflection is directly proportioned to $\overline{u_1 u_2}$, and in this way $\frac{\overline{u_1 u_2}}{\overline{u^2}}$ can be determined. Since, however, when y is very small and the correlation coefficient closely approaches unity, the determination of $1 - R^2$ by this method (on which the curvature of $R(y)$ depends, as previously stated, in relation to y for $y = 0$) is somewhat inaccurate. Taylor (reference 11) suggested the following interesting method for the direct measurement of $1 - R^2$: The vertices A, B and C, D of two Wheatstone bridges carrying two hot wires F_1 and F_2 are connected to two rheometric wires in the manner shown in figure 8; the two identical bridges are balanced by the mean resistance of the wires and so the medium tensions of A, B, C, and D are equal. At a point E on AB and at a point H on CD the predetermined tensions e_1 and e_2 are applied across a suitable amplifier to a milliammeter the readings of which are then proportional to $\overline{(e_1 - e_2)^2}$. Then $e_1 = \alpha E_1$; $e_2 = \beta E_2$ if E_1 and E_2 are the instantaneous potentials of A and B, and $\alpha \beta$ the ratios according to which the wires AB and CD are divided by E and by H. Hence the reading I on the milliammeter is

$$I = k(\alpha^2 E_1{}^2 + \beta^2 E_2{}^2 - 2\,\alpha\,\beta\,E_1 E_2)$$

On the other hand, since each of the two wires is suitably compensated,

$$E_1 = a\,u_1;\quad E_2 = a\,u_2$$

hence

$$Ry = \frac{\overline{u_1 u_2}}{u^2} = \frac{\overline{E_1 E_2}}{E_1{}^2},\quad \text{with}\quad \overline{E_1{}^2} = \overline{E_2{}^2}$$

so that

$$I = k(\alpha^2 \overline{E_1{}^2} + \beta^2 \overline{E_2{}^2} - 2\,\alpha\,\beta\,R\,\overline{E_1{}^2}) = k\,\overline{E_1{}^2}(\alpha^2 + \beta^2 - 2\,\alpha\,\beta\,R)$$

Consequently, if the correlation coefficient R between the fluctuations at the points where the wires are arranged is 1, it is possible to find for any point E taken on AB, another point H on CD so that by varying their potentials according to arbitrary law the meter, reading M is zero, provided that $\alpha = \beta$.

If R is not equal to unity I is minimum for $\beta = \alpha R$ and for this value of β, $I\,\text{min} = k\,E_1{}^2,\ \alpha^2(1 - R^2)$. On the other hand, if I_2 is the meter reading M when $\beta = 0$, or H coincides with C, $I_2 = k\,\alpha^2 E_1{}^2$ for which $1 - R^2 = \dfrac{I\,\text{min}}{I_2}$.

Determination of Correlation Coefficients between the Velocity Fluctuations in Non-isotropic Turbulent Flows

If the turbulence is not isotropic the number of quantities characterizing it increases considerably and the devices described in the foregoing are no longer practical, neither have there been found ways and means as far as I know, for an experimental solution of the problem. It should therefore be interesting to see how this selfsame instrument with several wires grouped to form the edges of a pyramid with a small vertex angle previously used for measuring the mean (vectorial) speed can lead, in conjunction with devices and equipment considered necessary for the study of turbulent flows, to the complete solution of the tensor of the turbulent flow, that is, the mean-square averages $\overline{u'^2}$, $\overline{v'^2}$, $\overline{w'^2}$ of the fluctuations between the three velocity components, and the average values of the products $\overline{u'v'}$, $\overline{v'w'}$, $\overline{u'w'}$.

Take, for the sake of simplicity, the case of two-dimensional turbulence for which it is sufficient to consider three single wires F_1, F_2, F_3 grouped so that plane F_1F_2 is orthogonal to plane $F_1 F_3$. Inserting F_1 in the usual manner (fig. 9) normal to air flow causes I_1 to give $\overline{u'^2}$. Then insert the two wires F_1 and F_2 in the arms of a Wheatstone bridge (fig. 10) and apply the difference of potential between the arms C and D, produced in consequence of the velocity fluctuations in plane F_1F_2 suitably compensated and amplified to a thermal-type milliammeter; the reading I_2 is directly proportional to the mean-square average $\overline{\alpha'^2}$ of the angular fluctuations of the direction of the instantaneous velocity with respect to its mean direction.

Hence follows $\overline{v'^2} = U^2\, \overline{\alpha'^2}$, where U is mean velocity.

Lastly, dispose wires F_1 and F_3 in series on one arm of the bridge. The variation of the resistance δr of the $F_1 F_3$ system by the small angle formed with the instantaneous direction of the wind is at every instant $\delta r = a\, u' + b\, v'$, where a and b are constants for each pair of wires, while an eventual component w' of the velocity fluctuation does not alter r if w' is small, since for it the resistance F_3 decreases as that of F_1 increases. Hence the reading on the milliammeter is given by

$$I = \overline{k(a\ u' + b\ v')^2} = k(a^2\ \overline{u'^2} + b^2 \overline{v'^2} + 2\ a\ b\ \overline{u'\ v'})$$

and with

$$I_1 = k\ \overline{u'^2}; \quad I_2 = k\ \overline{v'^2}$$

it affords

$$R = \frac{1}{2ab}\ \frac{I}{\overline{I_1}\ \overline{I_2}} - \frac{a}{2b}\ \overline{\frac{I_1}{I_2}} - \frac{b}{2a}\ \overline{\frac{I_2}{I_1}}$$

It should be interesting to note also that the thermal lag of the two-wire system, for example, F_1 and F_2 used in the determination of $\overline{v'^2}$, is still the same one corresponding to a single wire, as, for example, of F_1 used for measuring $\overline{u'^2}$. And, in fact, by this very method of Burgers (loc. cit.),

$$C\ \frac{d\theta_1}{dt} = 0.239\ i_1^2\ r_1 - Q_1(U,\ r_1,\ \alpha_1)$$

where C is the heat capacity of the wires assumed equal for both, and θ_1, r_1, i_1, Q_1 are, respectively, the temperature, resistance, heating current, heat transferred by F_1, and Q_1 obviously a function of the air velocity, temperature of the wire and of the inclination of U toward the wire.

Then, if r'_1 is the value of r_1 that should correspond to permanent conditions, the instantaneous values of U and of α_1 we get, supposing $r_1 - r'_1$ to be small of the first order:

$$C = \frac{d\theta_1}{dt} = -\ 0.478\ i_1 r_1\ \frac{\partial i_1}{\partial r_1}\ (r_1 - r'_1)$$

and analogously for F_2

$$C\ \frac{d\theta_2}{dt} = -\ 0.478\ i_2 r_2\ \frac{\partial i_2}{\partial r_2}\ (r_2 - r'_2)$$

which, after subtracting the first from the second and ignoring the small quantities of the orders above the first:

$$C \frac{d}{dt} (\theta_2 - \theta_1) = - 0.478 \; i \; r \frac{\partial i}{\partial r} [(r_2 - r_1) - (r_2' - r_1')]$$

or indicating

$$r_2 - r_1 = r; \quad r'_2 - r'_1 = r' \quad \frac{dr}{dt} = - \frac{1}{M} (r - r')$$

where M is again equation (7) and which is identical with (6).

It should be stressed again that the above possibility of measuring – by hot-wire anemometer – the average value $u'v'$ affords a useful check by the great complexity of the methods of measurement, of the operation of the anemometer itself in as much as the same value $u'v'$ in two-dimensional turbulence can be accurately and safely secured by simply measuring the pressure drop.

ELECTRICAL EQUIPMENT FOR MEASURING HYDRODYNAMIC ACTIONS

The present analysis is restricted to the electric equipment itself and specifically to one piece of equipment which is altogether new and original, namely, the electric dynamometer with condenser in its application to the prediction of constant forces and rotating forces.

Condenser-Type Electric Dynamometer

It is known that the forces exerted on a body in its motion relative to the flow are experimentally secured in wind-tunnel tests and afford the three components of the resultant force and of the moments. In aeronautical research it involves the determination of the aerodynamic actions not only on models stationary in the tunnel stream but also on models that rotate about an axis parallel to the direction of the air stream, thus reproducing the phenomenon of spin. Now, while the forces on a stationary model with fixed position in spcae can be measured with a normal weight balance, on a rotating body where the forces acting on it rotate with it, these forces cannot be equilibrated by a balance with axes fixed in space and hence vary continuously in position with respect to the forces.

The condenser-type dynamometer referred to was designed by the Turin Aerodynamics Laboratory for the purpose of defining the aerodynamic actions on airfoils and aircraft models rotating in the wind tunnel. The apparatus is also suitable for measuring the constant forces acting on a stationary model; although the weight balance has proved more satisfactory for this particular purpose. The electrical part of the dynamometer was designed and manufactured by the Allocchio-Bacchini company of Milan.

The principle on which the instrument is based is as follows: The force F to be measured, applied at P is transmitted, say by a cable, for example, to the plate A_1 of a condenser C, the other plate A_2 being solidly attached to a frame (fig. 12). The force F changes the state of tension, hence of the deformation of the metal plate which constitutes A_1 and consequently induces a variation in the capacity c of C, when the relative distance of A_1 and A_2 is changed. If F is constant, this variation of c remains constant also, but, if F rotates at angular velocity ω about P, the force at every instant transmitted by A_1 has the intensity $F \cos \alpha = F \cos \omega t$; hence it produces a sinusoidal variation of the distance between A_1 and A_2. Since A_1 is very rigid relative to the applied force, the variation in capacity at every instant can be kept approximately proportional to the variation of d, and so to F or $F \cos \omega t$.

The application of the above principle to the measurement of the aerodynamic forces has been used for several years by the French Aeronautical Research Section in the 1.80-meter wind tunnel at Paris (reference 16), but the realization of this principle as applied there permits only the measurement of constant forces for which, as a matter of fact, numerous other devices, simpler, more sensitive, and more accurate are available. The essential characteristic of the Turin condenser-type dynamometer, on the other hand, is that it enables measuring of the rotating forces. It is opportune in this respect to state that up to the present there is only one balance suitable for this purpose, and it is of the completely mechanical type; and that is of the 5-foot vertical tunnel of the NACA (reference 17), which is more complicated to use and to build, aside from certain serious difficulties of adaptation to a horizontal tunnel.

Operating Circuit

Figure 13 is a diagrammatic sketch of the dynamometer condenser C to which the constant or rotating force F is transmitted forms part of an oscillating circuit O_1 coupled loosely to a high frequency (10^6 cycles) generator O_2. The oscillations in O_2 excite oscillations in O_1. Under otherwise identical conditions the current in O_1, and hence the potential drop across C, is a function only of its capacity c, or of the distance d between A_1 and A_2. Then the diagram, the ordinate of which gives the potential drop V across A_1 and A_2 and the abscissa the capacity χ of O_1 has an aspect perfectly analogous to that of the resonance curve of circuit O_1 (fig. 14), which shows a maximum H at that value d_0 of d for which O_1 is under conditions of resonance excited by O_2; or, if X is the value of the capacity of O_1 for d = d_0 and L the inductance of O_1

$$c_0 = \frac{1}{{\omega_e}^2 L}$$

where ω_e is the pulsation of the oscillations in O_2. The diagram also shows two deflection points H_1 and H_2 for two values d_1 and d_2 of d, in proximity of which a suitably small given variation of d, $(d''_1 - d_1; d'_1 - d_1)$ obviously defines the maximum variation ΔV of the potential drop across A_1 and A_2. And since the resonance curve at these points can be likened to its tangent, the variations Δd of d define variations proportional to ΔV : $\Delta V = k\Delta d$.

The instrument is so designed that the capacity X of the circuit O_2 is the capacity c of the condenser C as well as the capacity c' of another variable condenser C' mounted in parallel; the latter is regulated in such a way that the total capacity, when no force is transmitted to C, is precisely equal to that at one of the deflection points of the resonance curve. This results in the twofold advantage of operating under conditions of maximum sensitivity (maximum k) and of securing a linear relation between the forces applied at C, which it is desired to measure, and the variation of the potential drop across the condenser plates which they produce.

The ΔV is applied, therefore, between grid and filament of the electron tube T. The grid of the latter is negatively charged; therefore, when O_2 does not operate and no oscillations are excited at O_1, no current is produced in the plate circuit of T. With V" and with V' are indicated further the amplitudes of the alternating EMF between the plates of condenser C which are produced when the distance d between them assumes the maximum and minimum admissible values of d''_1 and d'_1, the tension of the grid V_g resulting from that of the polarization and the superposed alternating tension, reaches a value approximating to zero, when the amplitude of the alternating tension is equal to V'' (fig. 15), while it reaches the value corresponding approximately to the lower corner of the characteristic dynamic grid plate of the tube when the difference in alternating potential applied to the tube is equal to V'. Then it is readily apparent that the anodic current I_a in the plate circuit of tube T will, in the two extreme cases considered, have the aspect of that of figure 15, whence the mean intensity of the said current will depend unequivocally, upon the capacity of the condenser C, and therefore on the force F applied to it. Suppose, for the present, that F is constant; then c will be constant also and the current in the anodic circuit of T will in successive instants have the form indicated in figure 16, that is, it will be represented by a family of curves of half waves, all positive and of equal amplitude and high frequency. In order to measure the mean value of said current, the tube T is mounted in series with a resistance R_1 (fig. 13) and in parallel with respect to two other resistances R_2 and R_3 hooked up in series, at the ends of which a constant potential drop H is applied. Thus, T, R_1, and R_2, R_3 form the arms of a Wheatstone bridge, while the other arm carries a high resistance voltmeter between points A and B. The values of the resistance and of the potential drop H are so regulated that the potential of A is equal to that at B when the current in the anodic circuit of T has the value of the capacity c of condenser C for $F = 0$.

Then, since the potential of B can be assumed constant, the potential drop across A and B, that is, the reading of voltmeter G_c, depends solely upon the variation of the mean potential of A and hence of force F. Now the variation ΔC of the capacity of C is proportional to F, while that of ΔV_g of the grid of T is proportional to ΔC; but that of the half waves which constitute their amplitude is such that their mean value is not proportional to ΔV_g and to F; however, the shifting of the relation of simple proportionality between $(I_a)m$ and F is small enough in the entire interval of the values of F for which the instrument is designed, so that no appreciable loss of sensitivity occurs.

If the force transmitted to condenser C is not constant, but varies according to the law $F' = F \cos \omega t$, the amplitude of the plate current in tube T varies also conformably to

$$I_a = I_m + a \cos \omega t \tag{14}$$

the mean value I_m of I_a depends upon the initial capacity of condenser C, while the variation of I_a from its mean value, that is, a, depends upon the amplitude of the variation of the condenser capacity and hence on F. In consequence, there is a potential drop D across the ends of resistance R_1, the amplitude of which is readily apparent from (14). D is further amplified and applied at an electronic voltmeter with maximum value, whose reading gives the maximum variation of I and hence depends solely upon the intensity F of F'.

Electric Circuit of the Dynamometer (fig. 18)

All the input circuits both of plate and grid and of the filaments of the electron tubes are in alternating current suitably transformed and rectified; appropriate regulating valves assure constant charging voltage in grid and plate, while a rheostat inserted in the feed circuit enables a constant value during the whole operation. The instrument is designed for six measuring condensers, a type herinafter indicated in the text with the symbol C_n. The condensers are connected to the instrument by shielded flexible cable. Each C_n is hooked in series to a small variable condenser M_n, so that it is possible, by proper adjustment of M before each test, to render the capacity of the circuits of the six metering condensers equal to one another. The insertion of each one into the detector circuit O_1 is simply then a maneuver with the meter (5).

In order to assure that detector O_1 in the absence of force F, has the capacity C_n corresponding to the deflection point on the resonance curve, a variable condenser C' hooked up in series with C_n and two small fixed capacities D_1 and D_2 in parallel, can be omitted or inserted by means of contacts (4) and (4'); D_2 is inserted and condenser C' regulated so that when inserted while D_1 is omitted, no change in the milliammeter reading occurs (6), (it indicates the plate voltage I_a in O_1). Under these conditions the total capacity of O_1 has a well-defined value X_1, which is obtainable from the resonance curve, by plotting the straight line parallel to the axis of the abscissas on which the curve itself cuts the segment representing the capacity of D_1. Now, if the fixed capacity D_2, which is exactly equal to $\chi' - \chi_1$ is omitted, the capacity of O_1 is so plotted that in each case it will have the value that corresponds to the deflection point on the resonance curve.

To assure time constancy in the dynamometer readings, it is obviously necessary that amplitude and frequency of the oscillations of O_1 correspond to the value χ_1 of its capacity to remain constant with respect to time. For this reason the oscillations are not produced directly in the circuit O_2 coupled loosely with O_1, but are controlled in O_2 by a booster generator O. Its oscillations with constant frequency assured by quartz crystal Q inserted in the grid circuit, are transmitted to O_2 across resonance amplifier O_3.

Varying with condenser 3, the degree of tuning between the two coupled circuits varies the ratio of amplification,thus makes it possible to keep the oscillations in O_1 constant and uniform as is necessary for the correct operation of the instrument.

For measuring forces the intensity of which ranges over a sufficiently large interval a change-over switch (7) is provided by means of which the resistances inserted between A and B (see fig. 13) are changed and respecting which millivoltmeter G_c in parallel measures the constant forces F applied at the condensers C_n. Another switch (8) changes the resistance R_1 where the potential drop of the plate current is measured and which, amplified and rectified, acts on the electrometer measuring the rotating forces.

Switches (7) and (8) can be set in three positions to which correspond, respectively, the minimum values of the measurable forces in the ratio 1 to 2.5 to 5. The absolute value of the force depends upon the rigidity of the plates in the condensers C_n to which the rotating forces F are transmitted. At present three types of condensers are provided: one for a maximum force of 2 kilograms, one of 5 kilograms, and one of 10 kilograms. Each condenser is designed as shown in figure 19: The two plates A_1 and A_2 are circular surfaces; A_2 is solidly attached to a cylindrical casing B which itself is bolted to a frame in the test chamber; A_1 is carried on a stem (or spindle) S insulated by sleeve D and connected to two steel plates E_1 and E_2 clamped across their periphery to box B by means of suitable flanges. Spindle S terminates at its upper end in a ring G at which the force to be measured is applied. This assures parallelism of the two flat condenser plates, no matter what the experiment to which the elastic plates are subjected.

Application of Dynamometer to Measurement of Aerodynamic Forces on Fixed or Rotating Models

If the model in the wind tunnel is fixed the solution of the aerodynamic actions presents no special procedure: Simply connect the six suspension wires of the model balancing the six components of the resultant forces and moments to the six condensers C_n in the manner indicated and proceed with the measurement. By this method it is possible to construct a six-component aerodynamic balance with direct reading of the forces. Its mechanical simplicity and the known advantages of the automatic equilibrium of the forces accruing with the condensers, coupled with the minimum displacement of the model during each test assured by the rigidity of the elastic plates E, are, however, opposed by the greater precision of the weight balance, the greater sensitivity which is almost independent of the capacity of the balance, and the certainty that all dangers of wrong calibrations are avoided.

For testing models in free or controlled rotation, the model is attached to a shaft A mounted in ball bearings S_1 and S_2 (fig. 20). The connection is by universal joint which permits the model to be disposed along any desired direction relative to the wind. The shaft A is along the tunnel axis and can either rotate freely about its axis or be controlled by flexible cable from an electric motor in the test chamber outside the fluid jet. The suspension of the shaft in the tunnel is indicated in figure 20. The ball bearing S_1 is mounted on two rigid streamline rods the ends of which are fastened to the tunnel walls and two horizontal bars, of which one s_1 is connected to a spring dynamometer one end of which attaches to a fixed point and the other s_2 terminates in metering condenser C_1. The suspension of bearing S_2 is identical, except that, while condenser C_2, to which S_1 is connected, is able to measure forces up to 10 kilograms, the condenser C_2, to which S_2 is connected, serves only for measuring forces up to 5 kilograms.

The spring dynamometers are intended to set up an initial tension in bar s_2 and s'_2 and render them capable of transmitting a compressive force; having considerably less rigidity than the plates E of the condensers C, the tension in s_2 and s'_2 can be kept constant and equal to the initial tension.

The system can be compared to a right-hand combination of fixed axes where axis x coincides in magnitude and direction with the tunnel axis as wind direction, y is vertical and positive toward the other, while origin O is the point of the axis resembling the support S_1. The angle φ defining the orientation of the system about x is measured by starting from the position for which the plane of symmetry of the model contains the direction of axis y; in this position y gives then the line of action of the <u>lift</u>; whereas z gives that of the <u>drift</u>. The components along x of the resultant force F_a and of the resultant moment M_a which the flow exerts on the model are constant for any orientation with respect to x, and can therefore be computed on the usual type of balance. The components of F_a and M_a in the plane yz, constant in intensity, rotate, however, about x with the same angular velocity ω of the model and therefore cannot be measured with the usual balance. With F and M denoting these particular components and φ_0 and φ_1, the angles formed by their lines of action with y at the initial instant, that is, for $\varphi = 0$, the force transmitted to condensers C_1 and C_2 at any instant t is:

On condenser C_1:

$$F \sin (\varphi + \varphi_0) + \frac{M}{a} \cos (\varphi_1 + \varphi) = F_2 \sin(\varphi + \varphi_2)$$

On condenser C_2:

$$-\frac{M}{a} \cos (\varphi + \varphi_1) = -F_1 \cos (\varphi + \varphi_1)$$

where a is the distance between the two supports s_1 and s_2, and F_2, F_1, φ_2 defined by

$$F_1 = \frac{M}{a}$$

$$F_2{}^2 = F_1{}^2 + F^2 - 2\ F\ F_1\ \sin(\varphi_1 - \varphi_0)$$

$$\cos\ \varphi_2 = \frac{F\ \cos\ \varphi_0 - F_1\ \cos\ \varphi_1}{F_2}$$

Now the readings on the galvanometer G_ν of the dynamometer inserted successively into the circuit of condenser C_1 and C_2 immediately afford the intensity of force F_1 and F_2, but not angles φ_1 and φ_2. To determine φ_1 and φ_2, that is, the phases of the sinusoidal forces that stress the condensers C_1 and C_2, two cylindrical sleeves carrying a threaded stem are fitted on the shaft normal to its axis and two weights m are disposed on it at a known distance d as shown in figure 20. Then, when the shaft and hence the model rotates, the aerodynamic forces F_1 and F_2 of known magnitude but unknown direction are superposed on the mass forces P_1 and P_2 likewise rotating at the frequency of F_1 and F_2 and known in intensity or in phase. With F'_1 and F'_2 denoting the intensity of the resultant forces read on the dynamometer after repeating the test under new conditions, the solution of the phases φ_1 and φ_2 follows immediately. Figure 22 presents the graphical solution for condenser C_2, by way of example, with OP_1 representing the vector which at instant $t = 0$ force P_1 produced on the condenser by the rotating masses m. The extreme point p of vector F'_1 originating in O and the resultant of P_1 and F_1 is readily obtained as intersection of two circles with centers in O and P_1, and radii equal to the known intensity F', and F'_1, which they defined by dynamometer.

Analytically we denote with ψ the phase of the force applied at P_1, and with φ_1 the phase of F_1 of the same triangle of figure 22 and get

$$F'_1{}^2 = F_1{}^2 + P_1{}^2 + 2\ F_1\ P_1\ \cos\ (\varphi_1 - \psi) \tag{17}$$

or

$$\varphi_1 = \psi + (\cos^{-1})\ \frac{F'_1{}^2 - F_1{}^2 - P_1{}^2}{2\ F_1 P_1} \tag{18}$$

and similar values for condenser C_1.

The forces of lift, drag, and moment with respect to axis y and axis z for $t = 0$ are obtained from

$$R_p = F_2 \cos \varphi_2 - F_1 \cos \varphi_1; \quad R_d = F_2 \sin \varphi_2 - F_1 \sin \varphi_1$$

$$M_y = F_1 a \sin \varphi_1; \quad M_z = F_1 a \cos \varphi_1 \tag{19}$$

Equations (17) and (18) enable the prediction of the maximum error committed in the determination of the phases φ_1 and φ_2 by the foregoing method and by the degree of accuracy attained in the dynamometer measurements; whence it is possible to deduce the necessary procedure for reducing the causes of error in the aerodynamic characteristics to a minimum.

To illustrate: Suppose that the relative maximum error possible in the determination of F_1 and F'_1 lies in equal measure in both as actually happens if the forces are of the same order of magnitude; that is,

$$\delta F'_1 = \pm \epsilon F'_1; \quad \delta F_1 = \pm \epsilon F_1 \tag{20}$$

Then the maximum error derived for phase φ_1 follows at

$$\delta \varphi_1 = \epsilon \frac{3 F'^2_1 + F_1^2 - P_1^2}{\sqrt{4 F_1^2 P_1^2 - (F'^2_1 - F_1^2 - P_1^2)^2}} \tag{21}$$

$$\delta \varphi_1 = \epsilon \frac{F'^2_1 - F_1^2 + P_1^2}{\sqrt{4 F_1^2 P_1^2 - (F'^2_1 - F_1^2 - P_1^2)^2}} \tag{22}$$

the first equation being valid if the errors at F'_1 and F_1 are of contrary sign, the second, when the errors are of the same sign. Substituting for $F'^2 - F_1^2 - P_1^2$, equation (17) and putting

$$p_1 = \frac{P_1}{F_1}$$

gives equations (21) and (22) written in the form

$$\delta \varphi_1 = \epsilon \frac{2 + p_1^2 + 3 p_1 \cos (\varphi_1 - \psi)}{p_1 \sin(\varphi_1 - \psi)} \tag{23}$$

$$\delta\varphi_1 = \epsilon \frac{p_1 + \cos(\varphi_1 - \psi)}{\sin(\varphi_1 - \psi)} \qquad (24)$$

From (23) and (24) it is readily apparent that it is possible either to cancel or at make $\delta\varphi$ a smaller fraction than ϵ, in every case, by putting $p_1 = 1$ and $\cos(\varphi_1 - \psi) = -1$; but under these conditions F'_1 would cancel also and the foregoing assumption relative to the value of ϵ would be far from true, because, even though $\delta F'_1$ cannot be canceled, the ϵ corresponding to it becomes infinite.

From (24) follows that for a given setting ψ of mass m the error $\delta\varphi_1$ is smaller as p_1 itself is smaller; but it is seen quite readily from (23) that then $\delta\varphi_1$ would be very large in the event of potential errors of contrary sign in the F_1 and F'_1 measurements. Hence - always referring to (24) - it is seen that $\delta\varphi_1$ can be canceled for a given value of p_1, less than unity by assuming $\cos(\varphi_1 - \psi) = -p_1$; but in that instance equation (23) becomes

$$\delta\varphi_1 = \epsilon \frac{2}{p_1}\sqrt{1 - p_1^2}$$

which affords for $p_1 = 0.7$, for example, a mere $\frac{F'_1}{F_1} = 0.6$ and $\delta\varphi_1 = 2.04\,\epsilon$, and even worse results when assuming values p_1 less than 0.7 in order to more closely approach the conditions

$$\frac{F'_1}{F_1} = \sim 1$$

On the other hand, securing the value of P_1 which makes $\delta\varphi_1$ a minimum for a given setting ψ, equation (23) gives

$$\frac{d}{dp_1}(\delta\varphi_1) = 0$$

$$[2\,p_1 + 3\cos(\varphi_1 - \psi)]p_1 - [2 + p_1^2 + 3\,p_1\cos(\varphi_1 - \psi)] = 0$$

hence,

$$p_1 = \sqrt{2} \qquad (25)$$

$\delta\varphi_1$ cancels when the errors ϵ are of contrary sign, by taking

$$\cos(\varphi_1 - \psi) = -\frac{2 + p_1^2}{3p_1} = -\frac{4}{3\sqrt{2}} = -0.945$$

we deduce under these conditions a maximum error of

$$\delta\varphi_1 = \epsilon\,\frac{1.41 - 0.945}{0.324} = 1.43\,\epsilon$$

while $\dfrac{F'_1}{F_1}$ is still smaller, equal to 0.58. For $p_1 = \sqrt{2}$,

the minimum error of $\delta\varphi_1$ that can be committed on the assumption that the measured ϵ's are of the same magnitude, we have when

$$\frac{d}{d\psi}(\delta\varphi_1) = -p_1\,\frac{\cos(\varphi_1 - \psi)}{\sin^2(\varphi_1 - \psi)} - \frac{1}{\sin^2(\varphi_1 - \psi)} = 0$$

whence

$$\cos(\varphi_1 - \psi) = -\frac{1}{\sqrt{2}} \tag{26}$$

that is, mass m must be set in such a manner that the action P_1 on condenser C_2 has a phase

$$\psi = \varphi_1 - (\pi + 45^\circ) \tag{27}$$

Under these conditions the error $\delta\varphi_1$ in both assumptions relative to the signs of ϵ, is always equal to ϵ, while F'_1 itself $= F_1$. Hence it is possible to predict, after one trail, a sufficiently approximate value of ω_1 for which equation (27) is always satisfied. But the application of (27) itself is rendered still easier because of the fact that the phases φ_1 and φ_2 of the aerodynamic forces applied at the two condensers are always quite small, so that if φ_1 is assumed to be zero in (27) the resulting value for ψ can be considered in maximum effect as a $\pm 7^\circ$ error with respect to that obtained with the effective value of φ_1. Then for

$$\varphi - \psi = \begin{cases} \pi + 38^\circ \\ \pi + 52^\circ \end{cases}$$

the corresponding values of $\delta\varphi$ read:

$$\delta\varphi = \begin{cases} 0.78\ \epsilon \\ 1.02\ \epsilon \end{cases} \text{ if the } \epsilon \text{ 's are of the same sign}$$

$$\delta\varphi = \begin{cases} 1.2\ \epsilon \\ \epsilon \end{cases} \text{ if the } \epsilon \text{ 's are of contrary sign}$$

always closer to ϵ, while for $\frac{F'_1}{F_1}$

$$\frac{F'_1}{F_1} \begin{cases} 1.12 \\ 0.9 \end{cases} \quad \text{always closer to 1.}$$

It may therefore be concluded that, if the masses are arranged in such a way as to satisfy (27) even if only approximately, and so disposed at a distance from the axis that the resulting value for F_1 is equal to $\sqrt{2}$, the F_1 obtained in the first measurement, the phases of the aerodynamic forces can be determined at least for a maximum error equal to ϵ; and, since ϵ is at maximum equal to 0.02, the extreme error in phase is $\pm 1^\circ$. Such an error, while negligible in a determination of lift and pitching moment, can lead to a much greater one in a measurement of the drag and of the moment about axis y as results from equation (19). However, this obstacle is readily removed by the following method: the mass m is placed for the initial position of the system in the plane yx, so as to quasi balance internally the aerodynamic action along axis y over both condensers, which is readily accomplished after having secured in a first test these same actions b virtue of the previously known values which have the phases φ_1 and φ_2 in each case.

By this method the resultant forces on the condensers are quite small and the dynamometer can be used by utilization of the maximum sensitivity of which it is capable; when proceeding under these conditions to define the intensity and phases of the residuary forces on the condensers themselves the previously found error of 1° is not made up on the phase of the total aerodynamic force which is quite small, but on the phase of the residuary force and since this is much greater, the relative error will be proportionally smaller.

In figure 20 the masses balancing the aerodynamic action along axis y are shown as solid lines, the smaller masses for measuring the phases of the residuary forces by dotted lines.

Some very satisfactory tests have been made so far with this equipment on a model wing in controlled rotation, the results of which are to be published in Rendiconti Sperimentali des Laboratorio di Aeronautica di Torino, series III.

Translation by J. Vanier,
National Advisory Committee
for Aeronautics.

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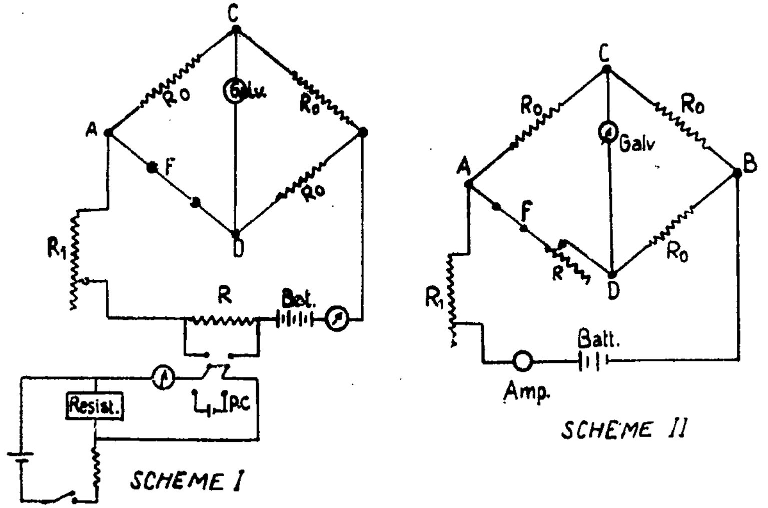
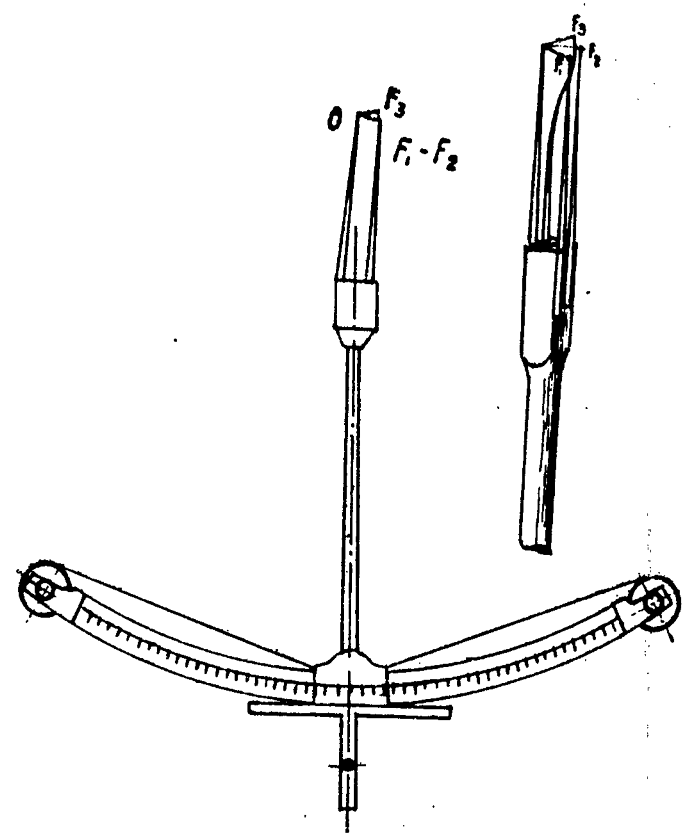


Fig. 1.



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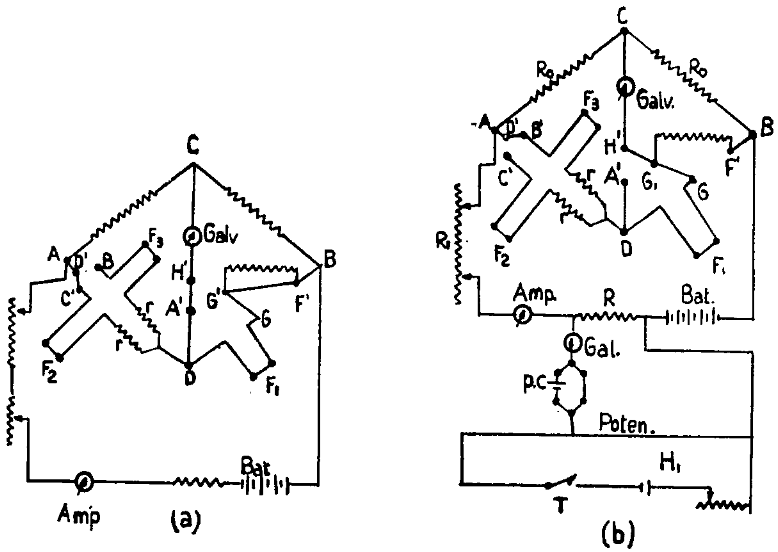

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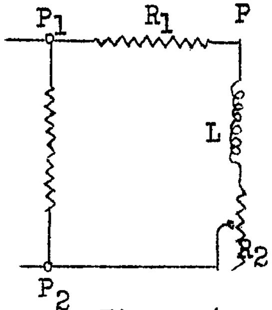

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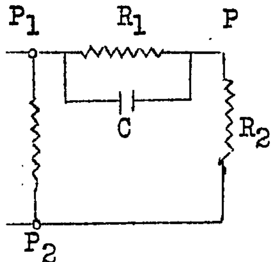
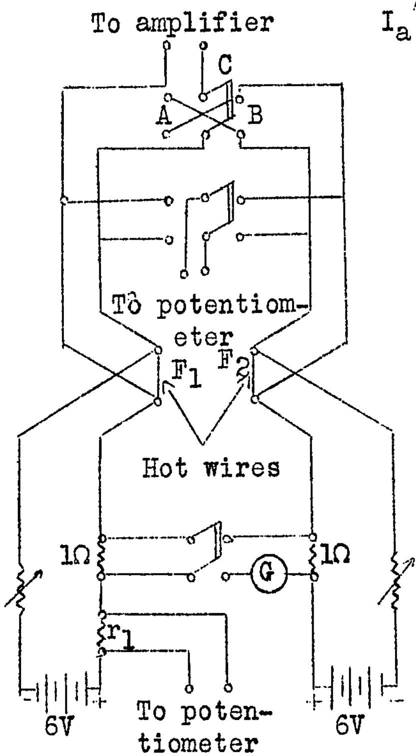


Figure 4a.



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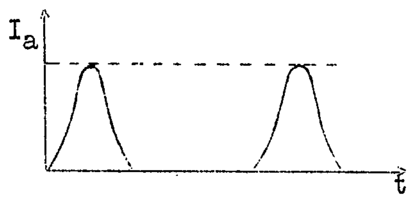

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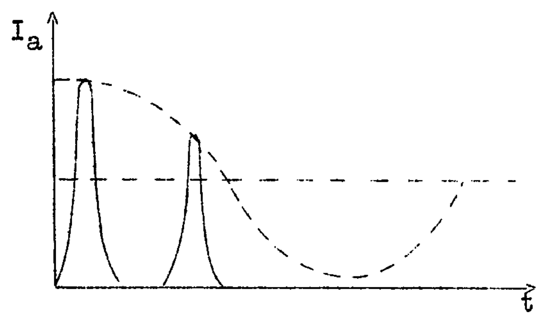

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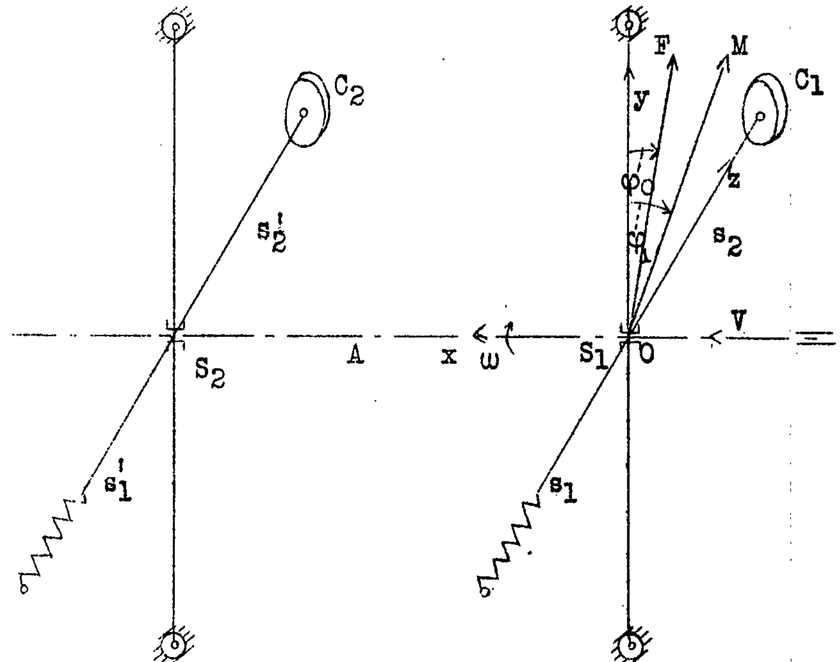

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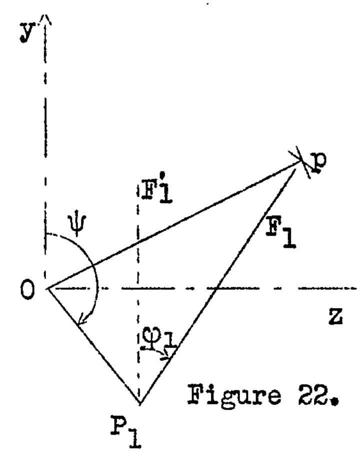

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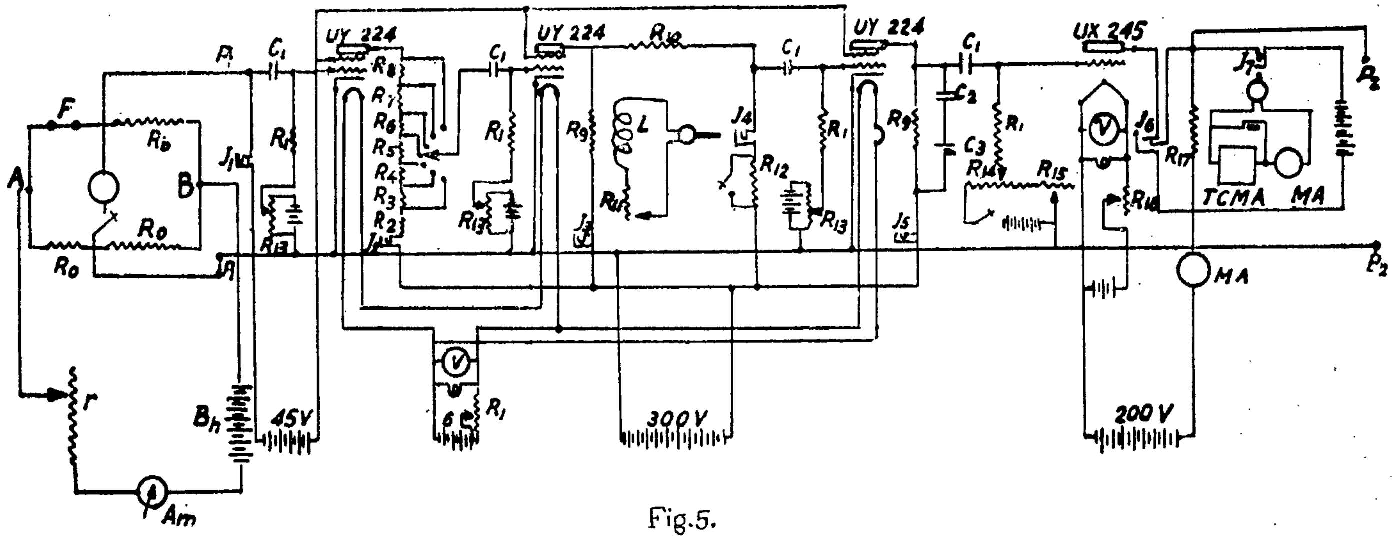
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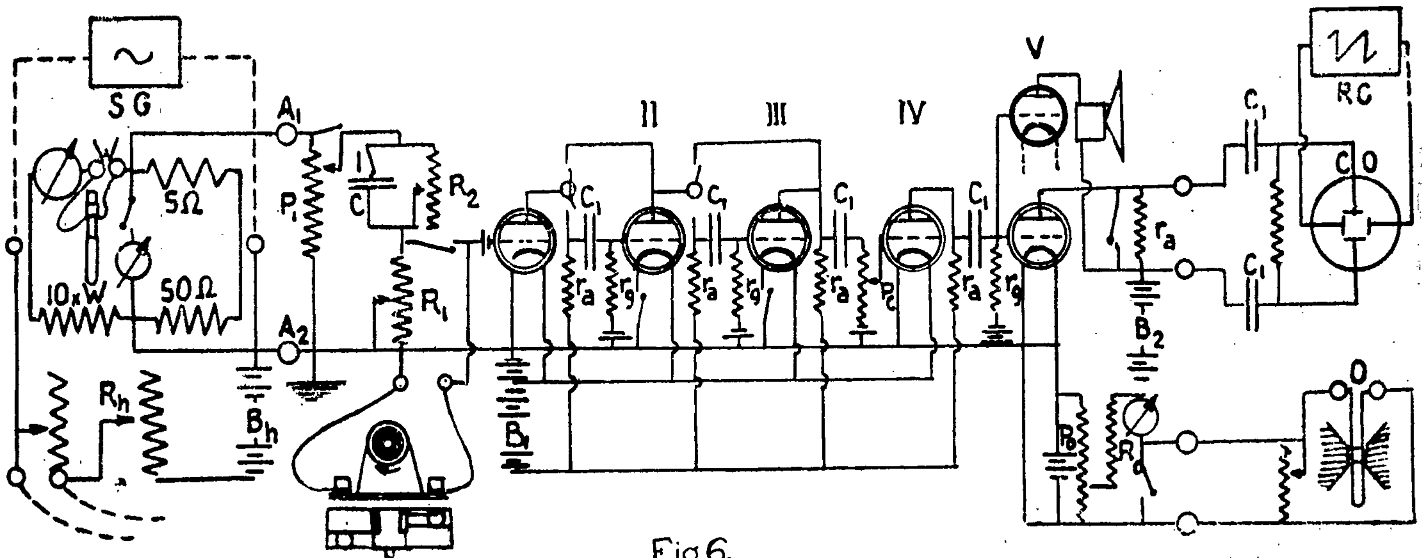
UY 224
UY 224
UX 245
45V
300V
200V
TCMA
MA
Am

Fig.5.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50Ω
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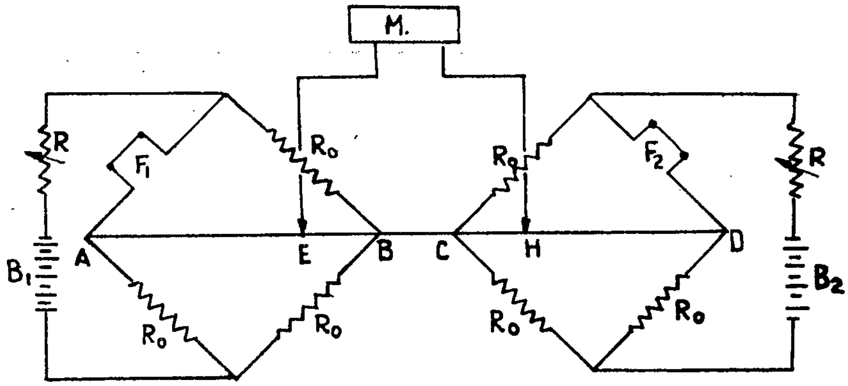
III
IV
V
RC
CO

Fig.6.



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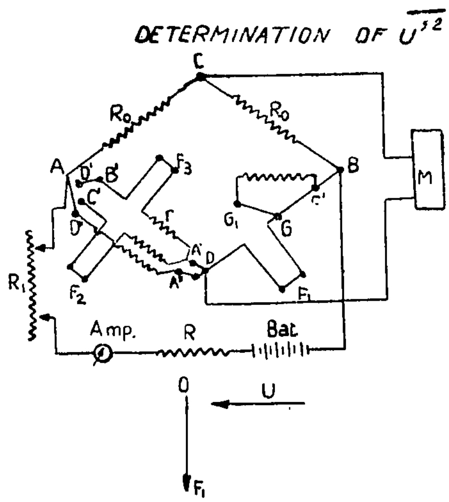
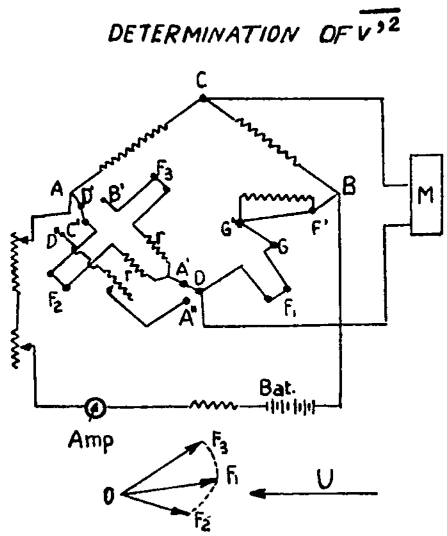


Fig. 9



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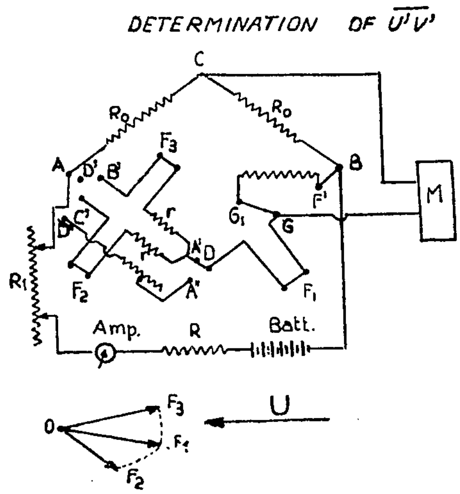

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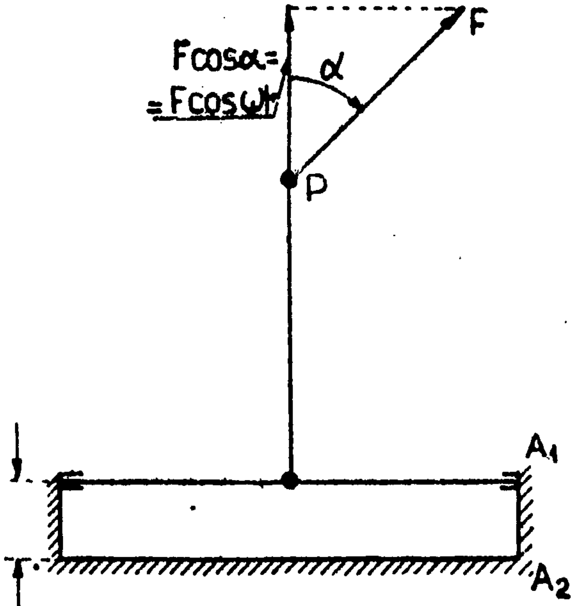
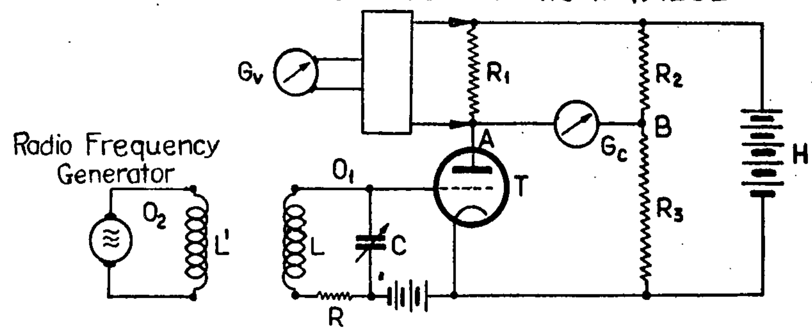


Fig.12.

ELECTRIC AMPLIFIER AND VOLTMETER FOR MEASURING MAXIMUM VALUES



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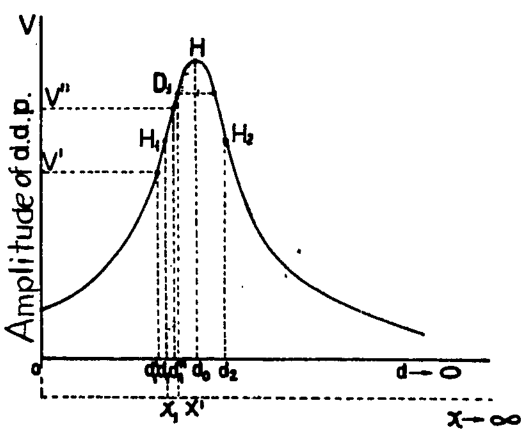

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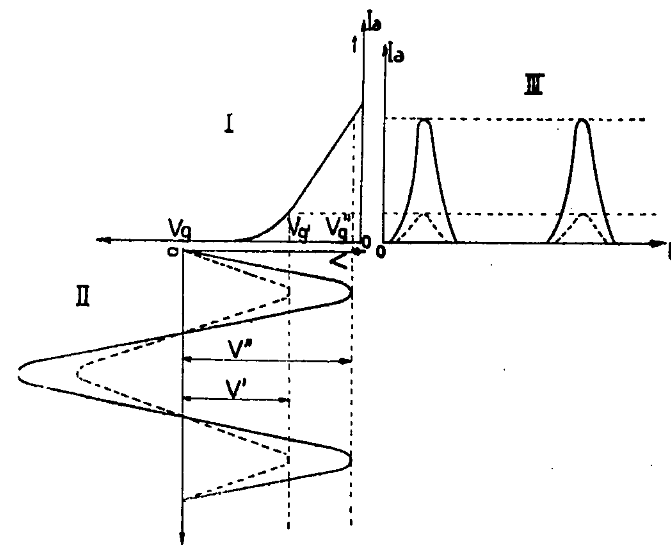

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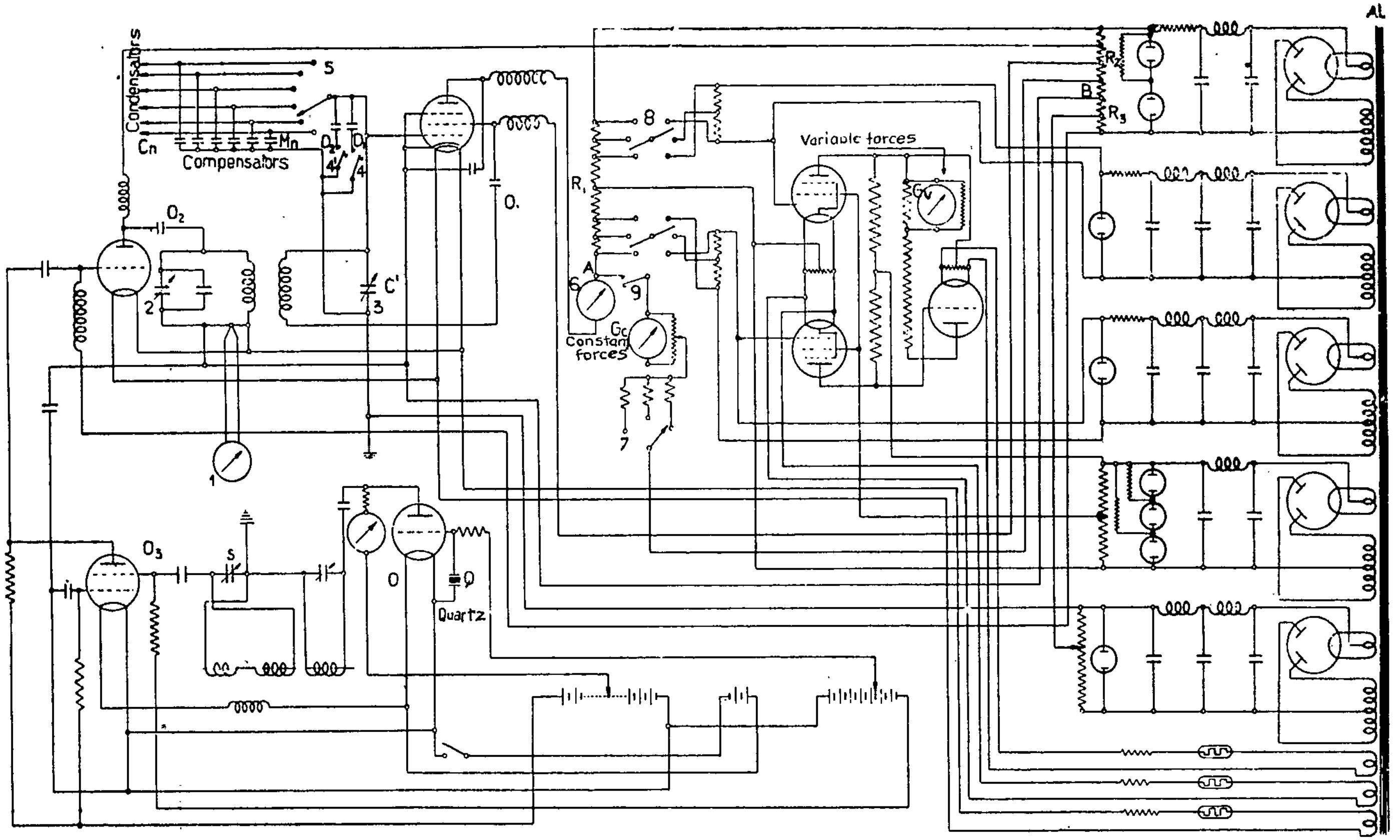

ELECTRICAL DIAGRAM OF
DINAMOMETER AND CONDENSERS

Fig. 18.

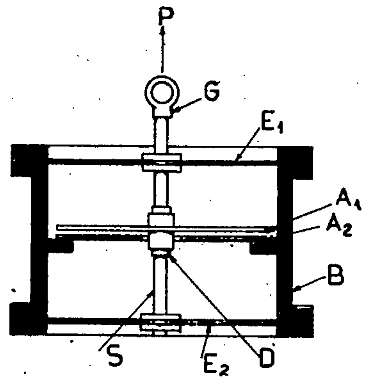

Fig. 19.

Fig. 20.

MECHANICAL DIAGRAM OF DINAMOMETER AND CONDENSERS